REGIME RELATIONSHIPS

Regime theory describes an approach to channel theory that assumes some form of equilibrium relationship between certain morphological parameters, such as width, or depth and hydraulic parameters such as hydraulic slope, discharge, or flow velocity. A summary of the range of relationships available has been drawn together by Spearman and these are briefly summarised in B3 (Spearman, 1995). Where sufficient historic data are available, these relationships can be used to explore aspects of the estuary development over time (ABP Research, 1999; ABPmer, 2003). Increasingly, however they are being used in conjunction with hydrodynamic models to create a form of hybrid model that can iterate to the equilibrium state (see Hybrid Methods).

Box 1 - Summary of regime variables

P	tidal prism (m^3)
A	cross-sectional area (m^2)
T	period of tide (s)
Q	discharge (m^3s^{-1})
W	width (m)
h	hydraulic depth of channel (m)
S	surface area (m^2)
V	volume (m^3)
v	velocity (ms^{-1})
s	energy slope
φ	lag between maximum slope and maximum velocity
With subscripts:	
max	at time of peak discharge
hw	high water
lw	low water
mtl	mean tide level

Table 1 - Summary of Regime Relationships (see Box 1 for definition of variables)[1]

Tidal Prism Relationships	
O'Brien (1931; 1969; 1972)	$A = C \cdot P^n$ where C and n are constants
Jarrett(1976)	$A = C \cdot P^n$ where $C=5.74 \times 10^{-5}$ and n=0.95
Eysink (1991)	$V = C \cdot P^{1.5}$ where C is an empirical constant that depends on the width, length, tidal range and shape of the estuary.
Hume and Herdendorf (1993)	Identify a range of types of estuary that form three distinct groups (see Townend, 2004).
Gao and Collins (1994)	$A = C \cdot P^n$ where $C=10^{-4}$ and $n \cong 1$
Kraus (1998)	$$A = \left(\frac{\alpha \pi^3 C_k^3 m^2 W_e^{4/3}}{Q_g T^3} \right)^{0.3} P^{0.9}$$ Where α and C_k are empirical coefficients close to unity, m is Mannings coefficient, W_e is the width corresponding to the equilibrium area, Q_g is the gross alongshore transport and T is the tidal period.
Hughes (2002)	$$A = 0.87 * \left[\frac{W^{1/9}}{\left[g(S_s - 1) \right]^{4/9} d^{1/3} T^{8/9}} \right] P^{8/9}$$ Where S_s is the sediments specific gravity, and the median grain size diameter, d.
Townend (2004)	Tests Kraus and Hughes relationships using UK data

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[1] All relationships are given in SI units. Many of the earlier references cited used imperial units and any dimensional constants have been converted accordingly.

Inlet Stability	
Brown (1928)	The maximum velocity is given by: $$v = Ch \cdot \left(\frac{A}{2pL}\right)^{1/2} \cdot \left(R^2 - r^2\right)^{1/4}$$ where Ch is the Chezy coefficient, p is the wetted perimeter of the channel, L is the length of the channel between the basin and the sea, R is the tidal range of the sea and r is the tidal range in the basin. The surface area of the bay is given by: $$S = \frac{C}{r} \cdot Ch \cdot A \cdot \left(\frac{A}{pL}\right)^{1/2} \cdot \left(R^2 - r^2\right)^{1/4}$$ where C is a constant.
Escoffier (1940)	Combining Brown's equations, Escoffier solved for velocity: $$v = Ch\left(\frac{AR}{2pL}\right)^{1/2} \cdot \left\{1/2\left(1 + f^2\right) - f\right\}^{1/2}$$ where $$f = \left(\frac{C \cdot Ch}{S}\right)^2 \cdot \frac{A^3}{2pHL}$$ The resulting maximum velocity as a function of inlet cross-sectional area can be compared with the equilibrium velocity (defined by some form of the O'Brien equation) to identify whether the inlet is likely to become unstable or not. This approach has been implemented as a pc utility Channel Equilibrium Area and can be downloaded from the US Army Corps web site. See also the report by Keulegan (1967) for a more extensive discussion.
Hughes (1999)	The equilibrium scour depth for non-cohesive sediments in tidal inlets is given by: $$h_e = \frac{0.234 \cdot q_e^{\ 8/9}}{\left[g \cdot \left(S_s - 1\right)\right]^{4/9} \cdot d_{50}^{\ 1/3}}$$ where h_e is the depth relative to the tide level at maximum discharge, d_{50} is the median grain-size diameter, S_s is the specific gravity of the sediment and q_e is the equilibrium maximum discharge per unit width. An online calculator for this relationship is available on the US Army Corps web site.
Form-based Tidal Asymmetry (see also guidance on flow based in Tidal Asymmetry Analysis)	
Dronkers (1998)	Using the hypothesis that morphological equilibrium equates to a uniform tide, Dronkers derived an asymmetry ratio based on certain estuary form parameters: $$\gamma = \left(\frac{h+a}{h-a}\right)^2 \cdot \frac{S_{lw}}{S_{hw}}$$ where h is the mean hydraulic depth of the estuary given approximately by $h=a+V_{lw}/S_{lw}$, although Roberts et al (1998) found that it was more reliable to use $h_{hw}=V_{hw}/S_{hw}$ and $h_{lw}=V_{lw}/S_{lw}$. The other variables are, a, the tidal amplitude, S_{lw}, the surface area at low water, S_{hw}, the surface area at high water, and V_{hw} and V_{lw}, the volumes at high and low water. Theoretically a value of γ equal to one suggests a uniform tide, with values greater than one indicating flood dominance and less than one indicating ebb dominance. In practice, Dronkers found that a value slightly greater than one provided a better indicator for Dutch estuaries and proposed a correction factor of 1.1.

Friedrichs and Aubrey (1988)	An alternative approach to asymmetry has been proposed by Friedrichs and Aubrey, based on a series of model simulations using a prismatic cross-section, they explored the influence of storage volume to channel volume, Vs/Vc, and the ratio of the M_2 tidal amplitude, a, to the hydraulic depth, h, (ie. a/h). They related variations in these two ratios to the amount of tidal distortion as identified from the ratio of M_4/M_2 amplitudes and the relative phase as given by $2M_2$-M_4 tidal constituent phases. Typically values of M_4/M_2 less than 0.01 suggest little tidal distortion or overtide. For the relative phase, values in the range $0<2M_2-M_4<180$ indicate flood dominance and when between $180<2M_2-M_4<360$ indicate ebb dominance.
Tidal Deltas	
Walton and Adams (1976)	$V_\delta = C \cdot P^{1.23}$ where V_δ is the volume of sand in the delta and C is a coefficient between x & y that varies with inlet exposure.
Sha and van den Berg (1993)	$\lambda_\delta = C \cdot P^n$ where λ_δ is the protrusion distance from the shoreline, C and n are constants, which for the Dutch coast are estimated to be 0.044 and 0.6 respectively (de Vriend *et al*. 1994).
Planimetric Relationships	
Renger and Partenscky (1974)	$\frac{S_{lw}}{S_{hw}^{1.5}} = 2.5 * 10^{-5} m^{-1}$ and $\frac{V_{lw}}{S_{hw}^{2}} = 8 * 10^{-9} m^{-1}$, which were derived from data for 22 tidal basins on the North Sea coast of Germany. Slightly different relationships have been derived using similar data for UK estuaries (see Townend, 2004).
Townend (2004)	$S_{mtl} = C \cdot P^n$ where C=0.42 and n=0.96
Hypsometric Relationships	
Boon and Bryne (1981)	$z_r = 1 - \left[\frac{s_x - s}{s_x \cdot r - s \cdot (r-1)} \right]^C$ $z_r = \frac{z - z_n}{z_x - z_n}$ $r = \frac{s_n}{s_x}$ Where subscripts x and n refer to the minimum and maximum values of elevation, z, and water surface area, s, and C is a constant. Recent work on estuaries (Townend, per com) suggests that this may be more appropriately expressed as: $z_r = \left(\frac{s}{a_x - s} \cdot \frac{a_x - s_x}{s_x} \right)^C$ where a_x is area of the estuary basin as a whole.
Wang *et al* (2002)	$V = V_o \cdot \left(1 + \frac{z}{d}\right)^{\gamma}$ from which it follows that $S = S_o \cdot \left(1 + \frac{z}{d}\right)^{\gamma - 1}$ where z is elevation, d is the depth of the channel and γ is a constant.
Discharge Relationships	
Keulegan (1951)	$Q_{max} = \pi CP/T$ where C is a constant between 0.81 and 1.0
Leopold and Maddock (1953)	$v \propto Q^m$; $W \propto Q^b$; $h \propto Q^f$; $s \propto Q^z$ where the exponents were found to have values of m=0.1, f=0.4, b=0.5 and z can vary between -0.5 and 1.0.
Langbein (1963)	$W = W_o \cdot \exp(-n \cdot \cot(\varphi) + k) \cdot x$ where W_o is the width at the mouth, n is a parameter dependent on $\sqrt{h}$

	and k is a constant. He used this, together with entropy arguments to suggest that the exponents in the discharge relationships should take on values of b=0.71, m=0.05, f=0.24 and z= -0.12
De Jong & Gerritsen (1984) Friedrichs (1995)	$A = Q \cdot Ch \cdot \left(\frac{\rho g}{\tau_s} \right)^{1/2}$ where A is the cross-sectional area, Q is the peak tidal discharge, Ch is the Chezy roughness coefficient (Friedrichs uses $Ch=h^{1/6}/n$, where n is Manning's friction coefficient and h is the channel depth), ρ is the fluid density and τ_s is the stability shear stress. The lower bound value of τ_s is provided by the critical shear stress $\tau_c \cong 0.6$Pa and the observed value of τ_s was found to vary with spring tidal range (R in metres) according to $\tau_s=2.3R^{0.79}\tau_c$.
Meander Relationships	
Langbein and Leopold (1966)	$\phi = \omega \cdot \sin\left(\frac{2\pi s}{M} \right)$ where ϕ is the direction at location s, ω is the maximum angle the meander takes relative to the valley direction and M is the channel length of a meander.
Deng and Singh (1999)	Use entropy arguments to show that $M=2\pi R$ and $\omega=90^{o}$ describes the stable equilibrium form, where R=meander radius.
Ferguson (1975)	$L = 57 \cdot Q^{0.58}$ where L is the wavelength and Q is the discharge exceeded 1% of the time.
Murray and Pethick (1999)	$a_m = 49.9 \cdot Q^{0.38}$ where a_m is the meander amplitude and Q is the mean tidal discharge.

The other use of regime relationships is to consider the implications of a change. If an area is to be reclaimed or dredged, then some of the gross properties of the estuary will be altered. Regime relationships can be used to determine whether the changes are likely to move the system towards or away from the particular equilibrium condition and whether or not the change is likely to be significant. This is illustrated in Box 2 for a development proposed on Southampton Water.

Box 2 - Example of use of Regime Relationships to assess changes due to a development

The table below lists values for a number of key form parameters (volumes, etc) for both historical and proposed cases, followed by selected regime relationships. The percentage change given is for the difference between the 'with terminal' and existing cases. Some of the regime relationships have typical or theoretical values as follows:

- O'Brien: $P/A = 1\text{x}10^4$ (based on the data compiled by (Gao & Collins, 1994))
- Renger: $V_{LW}/S_{HW}^2 = 8 \times 10^{-9}$ (Renger & Partenscky, 1974)
  $S_{LW}/S_{HW}^{1.5} = 2.5 \times 10^{-5}$
- Dronkers: $\gamma = 1$ for a symmetric tide (Dronkers, 1998)

As can be seen from the table below, the existing situation is not particularly close to any of these values. This is not surprising, as the reclamation to construct the existing port will have reduced the area and volume at high water, whilst channel dredging will have increased the area and volume at low water. What is however surprising, is the values in 1783, which are also some way from the equilibrium values. This might reflect the fact that the system has not yet reached an equilibrium state. Alternatively such differences might, at least in part, be a consequence of the highly distorted tidal wave that is delivered to the estuary because of its position in the English Channel.

Changes in regime relationships

Parameter	1783	1926	Existing	With Terminal	% Change
Hydraulic depth at HW (m)	5.15	5.23	7.28	7.60	+4.4
Hydraulic depth at LW (m)	6.09	5.11	6.32	6.88	+8.9
Tidal prism ($\text{x}10^8$ m³)	1.09	1.16	0.96	0.94	-1.5
Volume at LW ($\text{x}10^8$ m^3)	0.76	0.86	1.17	1.28	+9.4
Surface area at HW($\text{x}10^7$ m^2)	3.73	4.13	3.12	3.12	0
Surface area at LW($\text{x}10^7$ m^2)	1.22	1.74	1.90	1.90	+0.3
CSA at mouth ($\text{x}10^3$ m^2)	15.0	10.6	14.5	14.5	0
Regime Relationship					
O'Brien[1], prism/CSA ($\text{x}10^4$ m)	0.73	1.09	0.66	0.65	-1.5
F&A[1] friction, a/h	0.23	0.26	0.19	0.18	-6.1
F&A storage, V_s/V_c	0.46	0.34	0.15	0.14	-11.7
R&P[1], V_{LW}/S_{HW}^2 ($\text{x}10^{-9}$ m^{-1})	54.6	50.5	119	131	+9.5
R&P, $S_{LW}/S_{HW}^{1.5}$ ($\text{x}10^{-5}$ m^{-1})	5.36	6.54	10.9	10.9	+0.3
Dronkers gamma	0.23	0.44	0.81	0.74	-7.8

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